Reduction of the local Langlands correspondence to supercuspidal representations

In this talk we present several conjectures about a general reductive p-adic group G, and we explain them with examples. The ultimate aim of these conjectures is to decompose to problem of finding the local Langlands correspondence for G into: - the local Langlands correspondence for supercuspidal representations of Levi subgroups of G, - two other, presumably easier, problems.

On the representation side of the LLC we consider any Bernstein component in the space of irreducible (smooth, complex) G-representations We expect that it admits a strikingly simple geometric description, formulated in terms of a twisted extended quotient of the associated Bernstein torus with respect to the associated finite group. That this should be possible is motivated mainly by the shape of the Hecke algebras associated to a Bernstein component.

On the Galois side of the correspondence we observe a similar structure: enhanced Langlands parameters for G can be constructed from parameters for a Levi subgroup and twisted extended quotients.

In joint work with Anne-Marie Aubert, Paul Baum and Roger Plymen, all these conjectures have been proven for:

- inner forms of GL_n,
- inner forms of SL_n,
- principal series representations of split groups.